H = ratio of transfer coefficient to diffusivity
K_f = mass (heat) capacity of fluid (volumetric)
K_s = mass (heat) capacity of solid (volumetric)
N = rate of diffusion into single particle
Pe = Peclet group Vd/D_L
p, q = real quantities that correspond to the Fourier transform of the rate of diffusion into unit volume of solid following a surface impulse
R = radius of particle
Re = Reynolds Group, $dU\rho/\mu$
s = Laplace transform parameter
t = time
U = superficial velocity
V = interstitial velocity
w = Fourier transform parameter
z = coordinate
γ = defined by Equation (9)

Greek Letters

η = bed porosity
θ = defined by Equation (7)
μ = viscosity of fluid
ρ = density of fluid
σ = defined by Equation (8)
ϕ = number of particles per unit volume of solid phase

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Stirred-Tank Mechanical Power Requirement and Gas Holdup in Aerated Aqueous Phases

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Gas holdup and the ratio of the turbulent regime mechanical power consumed in aerated compared to that in nonaerated aqueous phases were measured in two laboratory sized tanks. Standard six-blade turbine ($D/T = 1/3$), six-blade paddle ($D/T = 1/3$), and four-blade paddle ($D/T = 2/3$) impellers were used over a wide range of impeller rotational speed and gas sparging rate. For all systems, the power ratio results were found to fit a semitheoretical correlation (derived from dimensional analysis) involving the impeller Weber number, the aeration number, and the ratio of dispersion and liquid densities. Empirical correlations for gas holdup in water, aqueous solutions of nonelectrolytes, and an aqueous electrolyte solution are given. The overall results lead to the conclusion that power ratio and gas holdup correlations are highly specific to a particular impeller type and are also dependent on the tank size and the liquid phase physicochemical properties.

SCOPE

Gas-to-liquid mass transfer often can be the rate limiting step in stirred-tank gas absorbers, chemical reactors in which one of the reactants is a dissolved gas, and in aerobic fermentation systems. Chemical and biochemical engineers, therefore, are frequently interested in predicting the rate of absorption of a gaseous species in a stirred-tank, gas-liquid dispersion, such predictions usually being based on correlations of the overall volumetric mass transfer coefficient (K_La) with mechanical agitation power per unit volume (P_g/V) and gas sparging rate expressed as the superficial velocity. Although no universally applicable correlation is yet available, if P_g/V is known the mass transfer characteristics of gaseous dispersions in water, organic liquids, or nonelectrolytic solutions (Calderbank, 1958, 1959; Calderbank and Moo-Young, 1961), or aqueous solutions of inorganic electrolytes (Lee and Meyrick, 1970; Robinson and Wilke, 1973) can at least be estimated.

The mechanical agitation power requirement of an ungassed Newtonian liquid (P_o) can be predicted for a number of impeller types from semitheoretical correlations of power number (N_P) with impeller Reynolds number (N_{Re}) as given, for example, by Rushton et al. (1950) and Bates et al. (1962). When a stirred liquid is sparged with a gas, the impeller power input to the gas-liquid dispersion decreases compared to that of the gas free

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liquid; that is, $P_g/P_o < 1.0$. The reduction in power is dependent upon the impeller type, tank geometry, liquid phase physicochemical properties, and gas sparging rate (Q). Only a limited number of empirical correlations are now available from which to estimate P_g/P_o, such as those of Ohyama and Endoh (1955), Michel and Miller (1962), and Pharamond et al. (1975). The first of these is of restricted usefulness as their correlating group, the aeration number $N_A = Q/ND^3$, does not uniquely characterize the Q-N interaction; the latter two are restricted to six-blade turbine impellers.

We have measured P_g/P_o and gas holdup for air sparged water and aqueous solutions of either inorganic electrolytes or organics in two fully baffled stirred tanks ($2.65 \cdot 10^{-3}$ and $19.0 \cdot 10^{-3}$ m^3 liquid volume) with three types of impellers (six-blade turbine and six-blade paddle, $D/T = 1/3$, and four-blade paddle, $D/T = 2/3$) over a range corresponding to a 100 fold variation in power input. The results have been correlated from a generalized equation deduced from dimensional analysis. We believe this correlating method results in improved accuracy in predicting P_g/V and, ultimately, K_La for mechanically agitated gas-liquid dispersions.

CONCLUSIONS AND SIGNIFICANCE

Dimensional analyses of liquid and gas-liquid dispersion mechanical agitation was used to derive a semi-theoretical equation which was found to represent the experimental data well. The impeller power ratio P_g/P_o in a turbulent, fully baffled system is described by an equation of the form

$$P_g/P_o = C_1 \cdot N_{We}^{m} \cdot N_A^{n} \cdot (\rho_L/\rho_D) \qquad (1)$$

For all aqueous phases tested, the exponent n was found to be constant regardless of impeller type ($n = -0.38$), whereas the fitted exponent m was slightly dependent on impeller type due, primarily, to differences in gas holdup among the three types of impellers. The data was best fitted by $m = -0.25$ for the six-blade turbine and paddle and by $m = -0.22$ for the four-blade paddle compared to $m = -0.19$ which is predicted for all impellers in the absence of different gas holdup effects. The constant C_1 was found to be dependent not only on impeller type and tank size (at constant impeller-to-tank diameter ratio) but also on the electrolytic nature of the aqueous phase.

For the mechanical agitation of turbulent liquids in fully baffled tanks, it is well known that P_o is proportional to N^3. Our results for mechanically agitated gas-liquid dispersions in the turbulent regime showed that for all practical purposes P_g also varied with N^3 for each of the three impellers used, except when the power input to the dispersion by isothermal gas expansion was substantial, that is, whenever $Q/V > 1/6$. The fact that P_g essentially is proportional to N^3 is consistent with the experimental results of Michel and Miller (1962), Rushton and Bimbinet (1968), and those of Pharamond et al. (1975) over a limited range of N.

No significant effect of tank size on gas holdup could be detected, but this may merely reflect the relatively small scale change used in our experiments. On the other hand, holdup was found to be dependent on impeller type and on the electrolytic nature of the aqueous phase, both of which affect C_2 and z in the general form of the equation describing gas holdup:

$$\phi = 1 - (\rho_D/\rho_L) = C_2(QN^2/\sigma)^z \qquad (2)$$

The presence of electrolytes in aqueous phase dispersions is known to significantly reduce the bubble average size and increase the gas holdup, thereby decreasing P_g. In such cases, the change in dispersion characteristics cannot be attributed to changes in interfacial tension (σ) but results from the coalescence inhibiting effects of ionic solutes. Therefore, correlations for water and aqueous solutions of nonelectrolytes cannot be used to predict (ϕ) and P_g in electrolyte solutions, and vice versa. In addition, numerical correlations for P_g appear to be highly specific with respect to both impeller type and scale; for example, considerable error may occur if P_g/P_o for a four-blade paddle ($D/T = 2/3$) is estimated from a correlation for six-blade turbines ($D/T = 1/3$), at least for laboratory sized stirred tanks.

THEORY AND PREVIOUS WORK

The interaction of pertinent system variables to produce a desired degree of turbulent regime mixing in liquids or gas-liquid dispersions cannot be predicted readily for a complex system such as a stirred tank for which complete hydrodynamic models are lacking. However, it is possible to relate the significant variables through dimensional analysis, thereby expressing system behavior in terms of various characteristic dimensionless groups.

Liquid Mixing Power (P_o)

Rushton et al. (1950) were the first to apply dimensional analysis in order to predict the dependency of the liquid mixing mechanical power requirement upon geometric, physicochemical, and dynamic parameters in stirred tanks. For specified geometry, P_o can be obtained from correlations of power number of the general form

$$N_P = P_o/\rho_L N^3 D^5 = C_3 N_{Re}^{x} N_{Fr}^{y} \qquad (3)$$

In fully baffled tanks, $y = 0$, and in the turbulent regime ($N_{Re} > 10^4$) Equation (3) reduces to

$$N_P = C_4 \qquad (4)$$

where C_4 is a constant specific for a particular impeller configuration. Correlations of N_P for various types of impellers have been given by Rushton et al. (1950) and Bates et al. (1962).

Gas-Liquid Dispersion Power (P_g)

It is well known that P_g cannot be determined from Equation (3) merely by substituting ρ_D for ρ_L (Blakebrough and Sambamurthy, 1964). That is, the reduction in mechanical agitation power drawn by a sparged compared to an unsparged liquid at constant N in a specified stirred tank does not depend merely on the change in the volume-average density of the tank contents resulting from the addition of dispersed gas bubbles.

We assume that the mechanical power required to agitate a gas-liquid dispersion depends on the impeller and tank geometry, the impeller rotational speed, liquid phase physicochemical properties, and the gas sparging rate. If we omit some geometric parameters (baffle width, T, etc.) for simplicity of illustration, the power may then be expressed in the following form:

$$P_g = C_5 N^a D^b \rho_L{}^c \mu_L{}^d g^e W_B{}^f V^h \sigma^m Q^n \rho_D{}^z \qquad (5)$$

If we apply dimensional analysis, Equation (5) can be rearranged to give

$$P_g/N^3D^5\rho_L = C_6(ND^2\rho_L/\mu)^d(N^2D/g)^e(W_B/D)^f$$
$$(\rho_L/\rho_D)^g(V/D^3)^h(N^2D^3\rho_L/\sigma)^m(Q/ND^3)^n \qquad (6)$$

For a fully baffled system ($e = 0$), a specific tank and impeller configuration, and for a system in the turbulent regime where mixing power is independent of viscosity ($d = 0$), Equation (6) simplifies to

$$P_g/N^3D^5\rho_L = C_7(\rho_L/\rho_D)^g(N^2D^3\rho_L/\sigma)^m(Q/ND^3)^n$$
$$= C_7(\rho_L/\rho_D)^g N_{We}{}^m N_A{}^n \qquad (7)$$

The dimensionless proportionality constant C_7 incorporates all the geometric parameters for a particular tank, that is, the impeller-tank relative dimensions and configurations.

The gassed-to-ungassed mechanical power ratio is obtained by combining Equations (4) and (7) to give

$$P_g/P_o = C_1 N_{We}{}^m N_A{}^n (\rho_L/\rho_D)^g \qquad (8)$$

Equation (8) forms the basis for correlating P_g/P_o with physicochemical properties (interfacial tension, density), impeller diameter and rotational speed, gas sparging rate, and gas holdup ($\phi = 1 - \rho_D/\rho_L$). Equation (8) has been derived for the case of nonelectrolytic aqueous phases; solution ionic strength does not appear in the correlation. In Equation (8), $C_1 = C_7/C_4$.

The aeration number (N_A) was first used by Ohyama and Endoh (1955) who measured P_g/P_o for air-water dispersions using a variety of impeller types: an eight-blade turbine, a paddle, and four types of vaned disks. In all of their runs, the impeller speed was held constant while the gas sparging rate was varied, the net effect being that P_g varied twofold. They found P_g decreased with increasing N_A (increasing Q at constant N); except for the turbine impeller in the range $0.04 \leqq N_A \leqq 0.12$, their plots of P_g/P_o vs. N_A were decidedly nonlinear and no simple algebraic correlation could be determined. Equation (8) shows that N_A alone is inadequate to describe the dependency of P_g/P_o on both of the primary operating variables in a prescribed gas-liquid system, the impeller rotational speed and gas sparging rate. Michel and Miller (1962) and Vares (1975) have determined that the change in P_g/P_o with varying N_A is not consistent but rather is dependent upon whether N_A is changed by altering Q or N. If Q is varied at constant N, continuously decreasing P_g/P_o vs. N_A functions similar to those of Ohyama and Endoh (1955) are obtained. However, if N is varied at constant Q, a separate curve is obtained for each gas rate, and these show P_g/P_o increasing slightly with increasing N_A until an asymptotic limit is reached such that P_g/P_o is constant, that is, no longer dependent on the impeller rotational speed. The limiting, constant value of P_g/P_o depends on Q, decreasing as Q increases.

Michel and Miller (1962) measured P_g for various liquids (water, organics, aqueous solutions of organics) in a $22.0 \cdot 10^3$ m^3 liquid volume tank (of nonstandard configuration) equipped with a six-blade turbine impeller ($D/T = 1/3$). Their results for this tank were correlated by the empirical relationship

$$P_g = C_8(P_o{}^2ND^3/Q^{0.56})^{0.43} \qquad (9)$$

They also found that all their data (two tanks, three D/T ratios) plus those of Sachs (1950), Ohyama and Endoh (1955), Bimbinet (1959), and Michel (1960) for six-blade turbine impellers could be represented within about $\pm 30\%$ by Equation (9) if the exponent 0.43 were changed to 0.45 and if $C_8 = 0.72$.

Clark and Vermeulen (1963) measured the power requirements for mixing gas-liquid dispersions with four-blade flat paddles and turbines of various sizes and impeller-to-tank diameter ratios. Water and organic liquids of different density, surface tension, and viscosity were used. As the Weber number was known to be a characteristic parameter for the mixing of two-phase media, it was used to correlate the power data. A geometrical group (G_v) incorporating impeller-tank dimensions developed from the study of vortex formation was also used as a suitable correlating factor for the different impeller sizes. The power ratio was given by Clark and Vermeulen (1963) as

$$P_g/P_o = f(\phi N_{We}{}^{1/4} G_v{}^{-1/2}) \qquad (10)$$

In this functionality, the Weber number was given an exponent of ¼ (the basis of this choice was not stated), while an exponent of $-\frac{1}{2}$ on the geometrical group was reported to give the best data fitting. Their experimental results showed that P_g/P_o decreased with increasing value of the function. The correlation of Equation (10) does not explicitly include the gas sparging rate. However, Figure 11 of Clark and Vermeulen (1963) indicates that the gas holdup can be considered to be a direct linear function of gas superficial velocity above sparging rates corresponding to $7.5(10^{-3})$ m/s.

Pharamond et al. (1975) evaluated P_g/P_o for air-water dispersions in three standard configuration tanks ranging in liquid volume from $18.7 \cdot 10^{-3}$ to $785 \cdot 10^{-3}$ m^3 and equipped with six-blade turbine impellers. Their results were correlated by the empirical equation

$$1 - (P_g/P_o) = 96(Q/V)D^{0.63} \qquad (11)$$

Equation (11) was applicable for $(Q/V)D^{0.63} < 0.3$; above this value, P_g/P_o was more or less constant at 0.5 to 0.55. It is implicit in Equation (11) that $P_g \alpha N^3$, a fact which Pharamond et al. (1975) observed for much of their experimental runs.

SYSTEMS INVESTIGATED

Stirred Tanks and Impellers

Two fully baffled stirred tanks were used in our studies. One was constructed from a piece of 0.152 m (6 in) I.D. Pyrex pipe, was equipped with stainless steel head plates and baffles, and contained a liquid volume of $2.65 \cdot 10^{-3}$ m^3; the design was similar to that of Robinson and Wilke (1973). The impeller shaft was driven by a variable speed electric motor; shaft rotational speed was continuously measured by an optical tachometer device connected to a digital read-out indicator. The motor was suspended above the tank from a wire and was equipped with air bearings as described by Calderbank (1958), thereby permitting the torque to be measured by means of a cord, pulley, and weighted scale-pan arrangement. Torque measurements in the liquids and gas-liquid dis-

Table 1. Stirred-Tank Geometry and Range of Operating Variables

Internal diameter, m	0.152		0.291	
Liquid volume				
—$m^3 \cdot 10^3$	2.65		19.0	
—h_L/T	1.0		1.0	
Air spargers				
—number	2		2	
—orifice diam., m	0.00317		0.00595	
—H_S/D	0.50		0.50	
—L_S/T	0.275		0.275	
Aeration rate, $m^3/s \cdot 10^3$	0.063-1.235		0.333-1.47	
Baffles—number	4		4	
—W_B/T	0.10		0.103	
Impeller type	$N(s^{-1})$	P_g/V (W/m^3)	$N(s^{-1})$	P_g/V (W/m^3)
Six-blade turbine	5-35	30-16 200	3.3-16.7	62-4 750
Six-blade paddle	5-35	30-19 000	3.3-13.3	27-3 470
Four-blade paddle	1.7-11.7	55-12 000	0.83-3.3	15-1 120

Table 2. Geometric Ratios of Impellers

Impeller type	N_B	D/T	W_I/D	L_I/D	T_I/D	H_I/D
Flat-blade turbine	6	1/3	0.20	0.25	0.03125	1.0
Paddle	6	1/3	0.20	1.0	0.03125	1.0
Paddle	4	2/3	0.188	1.0	0.020	1.0

Table 3. Physicochemical Properties of Aqueous Phases (at 25°C)

Liquid/solution	Density kg/m³	Viscosity ($N \cdot s/m^2$) $\cdot 10^3$	Surface tension (N/m) $\cdot 10^3$
Water	1 000	0.80	72.0
Propionic acid (9.8 wt %)	1 000	1.0	44.0
Methyl acetate (2.29 wt %)	1 000	1.0	58.2
Ethylene glycol (8 wt %)	1 008	1.27	55.0
Ethylene glycol (20 wt %)	1 021	1.72	47.4
Glycerol (40 wt %)	1 104	3.00	64.9
Sodium sulfate (0.4 kgmole/m³)	1 057	0.856	73.6

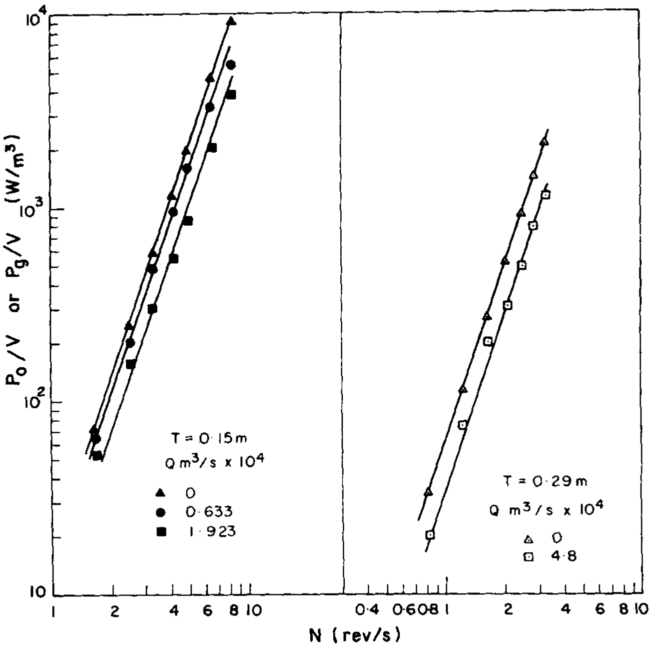

Fig. 1. Variation of mechanical power per unit volume with impeller rotational speed. Four-blade paddle ($D/T = 2/3$) in water and air-water dispersions.

persions were corrected for the no-load frictional torque, that is, the torque exerted by the impeller in air at the same rotational speed.

The second tank was constructed entirely of Perspex, was 0.291 m I.D., and contained $19 \cdot 10^{-3}$ m³ of liquid. A Chemineer Model EBL variable speed drive mixer turned the impeller shaft. The mixer drive assembly was free to rotate, being mounted on ball bearings; torque could be measured, therefore, by the same method used for the smaller tank. Rotational speed was measured with a stroboscope.

The air flow to each tank was measured by means of calibrated rotameters. Both tanks contained two single-hole orifice spargers fitted diametrically opposite through the bottom head plate and located 0.275 tank diameters from the center line of the tank. The sparger orifices in each tank were sized to give approximately the same orifice gas velocity in each tank at any given Q/V. Relevant dimensions are given in Table 1.

There is, apparently, no effect of sparger design on the gas-liquid dispersion characteristics in tank sizes similar to those used in this work as experimentally confirmed and reported by previous workers (Westerterp et al., 1963; Valentin, 1967). However, this is not likely to be the case for large-scale commercial tanks, where a gas distributor such as a ring-type sparger would no doubt yield a more-uniform distribution of the sparged gas than one or even two orifice spargers, especially at low impeller speeds.

Three types of impellers were used with each tank: a six flat blade turbine ($D/T = 1/3$), a six-blade paddle ($D/T = 1/3$), and a four-blade paddle ($D/T = 2/3$). The geometrical ratios for these impellers are given in Table 2, and the range of rotational speeds, aeration rates, and the resultant P_g/V for each impeller-tank combination are listed in Table 1.

The fractional gas holdup was evaluated from the difference in height above tank bottom between the gas-liquid dispersion and the ungassed liquid as measured by a cathetometer directed at the impeller shaft region, thereby avoiding the fluctuations in level that occur at the tank wall due to the baffles. The gas holdup was computed from

$$\phi = 1 - (\rho_D/\rho_L) = (h_D - h_L)/h_D \qquad (12)$$

All runs were done at room temperature, with tank head spaces at local barometric pressure.

Aqueous Phases

Power measurements were made in deionized water and aqueous solutions of propionic acid (9.8 wt%), methyl acetate (2.29 wt%), ethylene glycol (8 and 20 wt%), glycerol (40 wt%) and sodium sulfate (0.4 kg mole/m³). All chemicals were reagent grade. The pertinent properties of the various liquid phases are given in Table 3.

RESULTS AND DISCUSSION

Power Ratio

The impeller power input to the gas-liquid dispersions was found to be proportional to the cube of the impeller rotational speed, regardless of impeller type and tank size, when the gas sparging rate was held constant. This behavior is illustrated for the four-blade paddle in water on Figure 1, wherein it may be observed that P_g decreases as Q increases at constant N. Similar plots for the other two impellers and other aqueous systems gave the same dependency, that is, $P_g \alpha N^3$ at constant Q. This has previously been reported for six-blade turbines by Rushton and Bimbinet (1968) and for four-blade turbines by Moritz et al. (1974). The fact that $P_g \alpha N^3$ can also be concluded from the work of Michel and Miller (1962) with six-blade turbines; their empirical correlation, Equation (9), shows that $P_g \alpha (N^7)^{0.43} = N^{3.01}$. Therefore, in Equation (7), the term $P_g/N^3D^5\rho_L$, the gassed system power number, is a constant in the turbulent agitation regime for a specified gas sparging rate in a particular system (specified liquid phase, impeller-tank configuration).

Since both P_g and P_o are proportional to N^3, the power ratio P_g/P_o is independent of N; for a particular system, P_g/P_o is a constant at a specified gas sparging rate within a fairly wide range of Q. Cross plots of all the data in the form of Figure 1 showed that P_g/P_o varied as $Q^{-0.38}$; that is, the aeration number exponent in Equation (1) or Equation (8) is $n = -0.38$.

Preliminary regression analysis of the experimental P_g/P_o data for all systems studied to the function given by Equation (8) indicated that the dependency on the density ratio ρ_L/ρ_D was described by $0.95 \leqq g \leqq 1.0$. The variation in this computer fitted exponent value apparently is due to the differences in gas holdup observed with the various impeller types at otherwise constant operating conditions. For simplicity, it was decided to set exponent $g = 1.0$ for all impeller-aqueous phase systems and to let the effect of intersystem variations in gas holdup be reflected in the remaining exponent m of Equation (1). Since P_g/P_o is independent of N at constant Q Equation (8) shows for $n = -0.38$ and $g = 1.0$ that m should be -0.19 regardless of impeller type and scale. As discussed later, fixing exponent g at one value for all impellers used results in the best-fit experimental value of m for a particular impeller differing somewhat from -0.19.

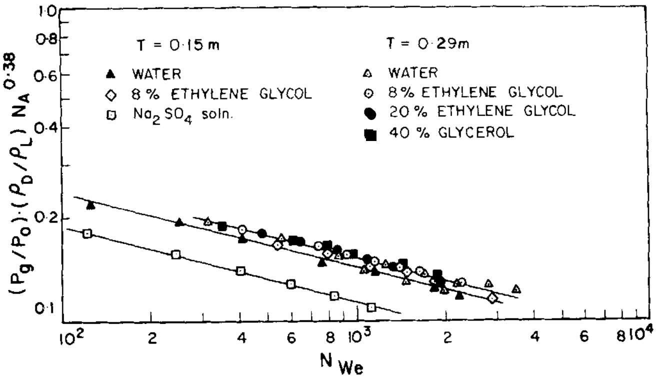

Fig. 2. Variation of power function with impeller Weber number. Six-blade turbines ($D/T = 1/3$) in water, aqueous solutions of nonelectrolytes, and sodium sulphate solution gas-liquid dispersions.

In cases where the power imparted to the gas-liquid dispersion by isothermal expansion of the sparged gas (from the sparger exit pressure to the head space pressure) P_q is small compared to the mechanical agitation power, that is, sufficiently low gas sparging rate per unit liquid volume such that $Q/V \leqq 1/6$, it was observed that $(P_g/P_o)\cdot(\rho_D/\rho_L)\cdot(N_A)^{0.38}$ was essentially constant, regardless of Q, at a constant impeller rotational speed (constant N_{We} for a particular system). Accordingly, an average of the values of $(P_g/P_o)\cdot(\rho_D/\rho_L)\cdot(N_A)^{0.38}$ obtained at each sparging rate with N constant was plotted vs. N_{We} in order to determine the value of exponent m [Equation (1)] for each system studied. Typical cases of the near constancy of $(P_g/P_o)\cdot(\rho_D/\rho_L)\cdot(N_A)^{0.38}$ for sufficiently small Q/V, the averaging method, and the variation in function value with increasing P_q/P_g are illustrated in Table 4. The results for all systems showed that the function $(P_g/P_o)\cdot(\rho_D/\rho_L)\cdot(N_A)^{0.38}$ was not essentially constant, and, therefore, it was inappropriate to use an average value over a range of Q in the following circumstances: $N \leqq 20$ with $P_q/P_g > 0.10$ and $N > 20$ with $Q/V > 0.1833$. Data obtained in such cases are not included in the correlations presented later.

TABLE 4. VARIATION OF THE FUNCTION WITH GAS SPARGING RATE. TYPICAL RESULTS FOR SIX-BLADE TURBINE WITH AIR-WATER DISPERSIONS IN 0.15 m I.D. TANK

N rev/s	Q m^3/s $\times 10^4$	N_{We}	P_q/P_g	$(P_g/P_o)\cdot(\rho_D/\rho_L)\cdot(N_A)^{0.38}$
11.667	0.633	248.1	0.0579	0.1838
	1.288		0.1385	0.1912
	1.923		0.2512	0.1933
35.00	0.633	2231.0	0 0020	0.1098
	1.288		0.0046	0.1105
	1.923		0.0078	0.1081
	3.400		0.0185	0.1046
				Mean 0.1083
				S. dev. 0.003
	4.834		0.0295	0.0956
	6.250		0.0415	0.0955
	7.584		0.0533	0 0851
	8.917		0.0658	0.0792
	10.12		0 0797	0.0724
	11.27		0.0907	0.0728
	12.35		0.1106	0.0722

TABLE 5. CONSTANTS FOR GAS HOLDUP AND POWER RATIO CORRELATIONS

$$\phi = 1 - \left(\frac{\rho_D}{\rho_L} \right) = C_2 \left(\frac{Q\,N^2}{\sigma} \right)^z ; \frac{P_g}{P_o} \cdot \frac{\rho_D}{\rho_L} \cdot$$

$$N_A{}^{0.38} = C_1\ (N_{We})^m$$

Impeller and aqueous system	C_2	z	Liquid volume m^3 $\times 10^3$	C_1	m
Six-blade turbine					
Water and aqueous nonelectrolytes	0.113	0.57	2.65	0.774	−0.25
			19.00	0.82	−0.25
0.4 kgmole/m^3 Na_2SO_4 solution	0.209	0.44	2.65	0.587	−0.25
Six-blade paddle					
Water and aqueous nonelectrolytes	0.102	0.65	2.65	0.80	−0.25
			19.00	1.20	−0.25
Four-blade paddle					
Water and aqueous nonelectrolytes	0.316	0.5	2.65	0.31	−0.22
			19.00	0.76	−0.22

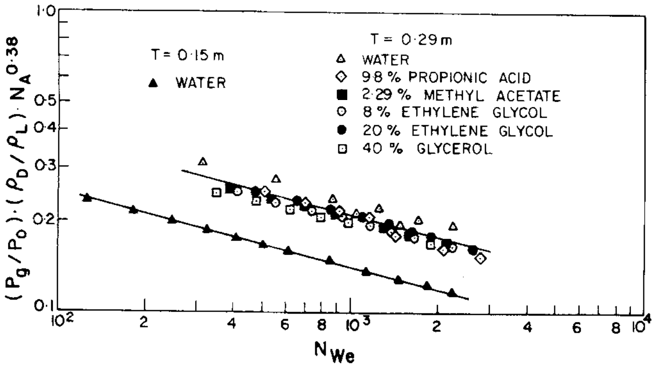

Fig. 3. Dependency of power function on impeller Weber number for six-blade paddles ($D/T = 1/3$) in aerated water and aqueous solutions of nonelectrolytes.

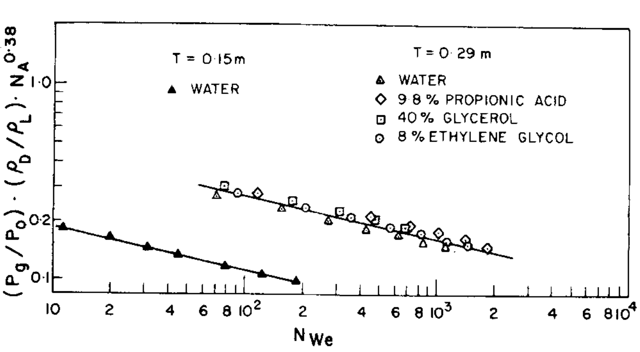

Fig. 4. Power function vs. impeller Weber number for four-blade paddles ($D/T = 2/3$) in aerated water and aqueous solutions of of nonelectrolytes.

Data for the six-blade turbine are plotted in the form of Equation (1) on Figure 2. Least-squares fitting shows that the data for all the aqueous systems and for both tank sizes can be represented well by lines of the same slope, that is, $m = -0.25$. However, it should be noted that there is a slight but nonetheless noticeable effect of tank size in systems comprised of water or aqueous solutions of nonelectrolytes. This difference is reflected in the proportionality constant (C_1), which incorporated all the geometric ratio factors resulting from the dimensional analysis, being dependent on the system scale as shown in Table 5. There appears to be no appreciable effect of tank size on gas holdup in water or aqueous solutions of nonelectrolytes (Figure 5), but at constant N_{We} and N_A the ratio P_g/P_o for these systems appears to decrease somewhat as the tank size is decreased. Further investigation over a considerably wider range of tank sizes is required before the effect of scale can be determined with confidence.

For the six-blade turbine in the smaller tank, Figure 2 also shows that significantly different P_g/P_o values were obtained in the aqueous solution of sodium sulfate compared to those for nonelectrolytic liquids. The power ratio for the electrolyte solution ranged from about 80 to 90% of that for water in the same size tank and at the same agitation-aeration rate conditions. This reduction in P_g/P_o for electrolyte solutions results from but is not directly proportional to the significant increase in gas holdup compared to that of water as shown by Figure 5. These observed differences in gas holdup cannot be attributed to any significant degree to differences in liquid phase density, viscosity, or surface tension between the electrolyte solution and water as shown by Table 3. However, it is well known that ionic solutes inhibit bubble coalescence, resulting in decreased bubble average size and increased gas holdup compared to water.

As an ionic strength parameter is not included explicitly in Equation (1), it is not expected to be generally applicable to aqueous solutions of electrolytes. However, it is interesting to note that the sodium sulfate solution data on Figure 2 are fitted by a line of the same slope as for water and nonelectrolytic aqueous solutions. Therefore, Equation (1) adequately represents the effects of N_{We} and N_A on P_g/P_o for electrolyte solutions. The effect of ionic strength on P_g/P_o for the sodium sulfate solution has, in effect, been incorporated into the proportionality constant C_1; for this case, then, C_1 does not merely reflect geometric parameters. Obviously, there is scope for further research into the effect of ionic strength on P_g/P_o in order to modify Equation (1) and make it completely general.

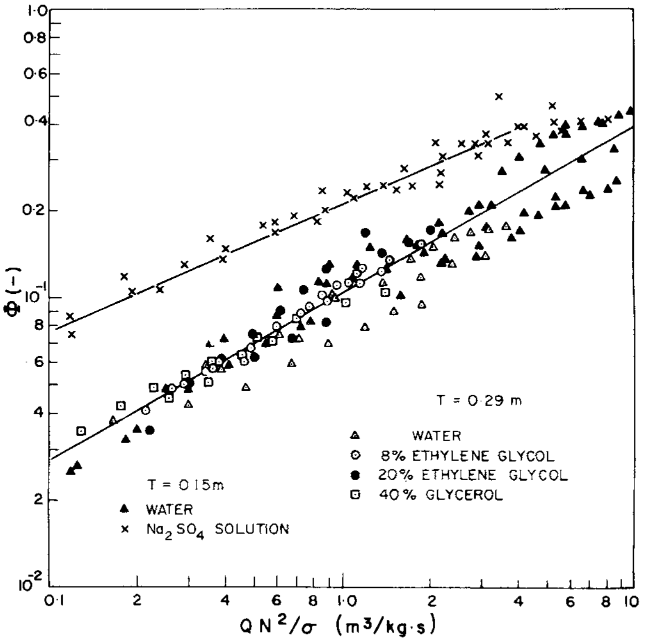

Fig. 5. Gas holdup with six-blade turbines ($D/T = 1/3$) in air sparged water, aqueous solutions of nonelectrolytes, and sodium sulfate solution.

Power correlations for the six-blade and four-blade paddles are given on Figures 3 and 4, respectively. For water and aqueous solutions of nonelectrolytes, $m = -0.25$ for the six-blade paddle and $m = -0.22$ for the four-blade paddle regardless of tank size. For both paddles in aerated water, P_g/P_o is significantly less in the smaller of the two tanks at constant N_{We} and N_A. The reduction in P_g/P_o with decrease in tank size for both paddles is greater than that observed for the six-blade turbine (Figure 2). For the six-blade paddle, the reduction in power ratio with decrease in tank size is 33 1/3% and for the four-blade paddle 59%, compared to a maximum of only 5.6% reduction for the six-blade turbine with nonelectrolytic aqueous phases. The change in P_g/P_o with tank size for the paddle impellers at constant N_{We} and N_A does not appear to be due to any significant change in gas holdup with tank size as shown by Figures 6 and 7.

The dependence of power reduction on surface tension for water and aqueous organic solutions is given through the Weber number; the best-fit N_{We} exponent in this work was found to range from -0.22 to -0.25, depending on impeller type. This is in good agreement with the N_{We} exponent of -0.25 obtained by Clark and Vermeulen (1963) for $\phi N_{We}^{1/4} G_v^{-1/2} > 1.8$. The direct use

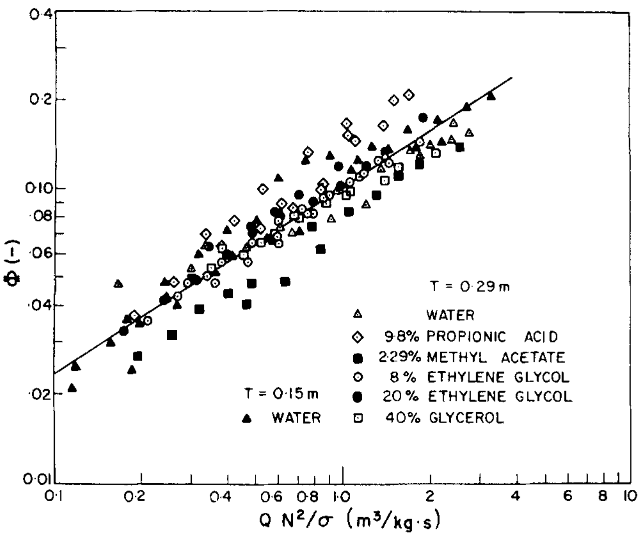

Fig. 6. Gas holdup with six-blade paddles ($D/T = 1/3$) in air sparged water and aqueous solutions of nonelectrolytes.

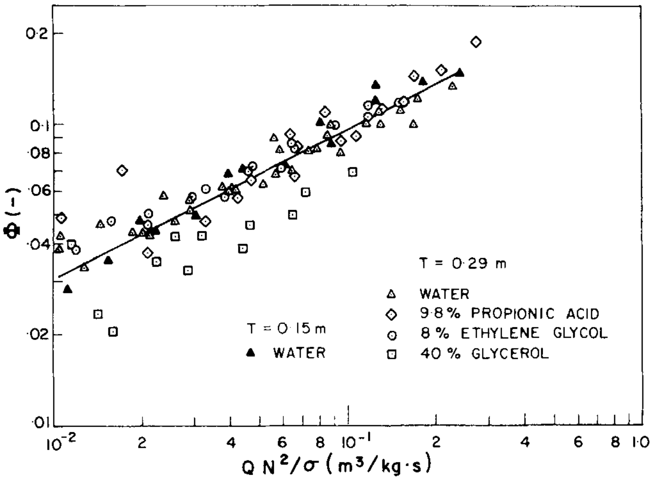

Fig. 7. Gas holdup with four-blade paddles ($D/T = 2/3$) in water and aqueous nonelectrolyte solution air-liquid dispersions.

of Equation (10) to estimate P_g requires a priori knowledge of ϕ. However, their final correlation for gas holdup also included P_g as a parameter. Therefore, the application of their correlations requires an iterative calculation procedure, whereas our proposed method does not. Moreover it must be noted that our data from nonelectrolyte systems with the four-blade paddle agrees with the general correlating function of Clark and Vermeulen, while those for six-blade impellers do not. This restricts the applicability of their correlation to systems using four-blade impellers (which were the basis of its development).

Gas Holdup

The change in gas holdup with solution type, gas sparging rate, and impeller rotational speed in all systems can be correlated by

$$\phi = 1 - (\rho_D/\rho_L) = C_2(QN^2/\sigma)^z \qquad (2)$$

where both C_2 and z are dependent on impeller type and the electrolytic nature of the aqueous phase but are not dependent on tank size. The change in ϕ with QN^2/σ is shown on Figure 5 for the six-blade turbine, Figure 6 for the six-blade paddle, and Figure 7 for the four-blade paddle.

For the six-blade turbine, Figure 5 illustrates the significant change in gas holdup (ranging from 70 to 175% increase, except at the highest values of QN^2/σ studied) that results from the addition of the ionic solute to water. This range of increase in gas holdup results in a corresponding decrease in ρ_D/ρ_L ranging from 5 to 16%. However, at constant N_{We} and N_A, Figure 2 shows that $(P_g/P_o)\cdot(\rho_D/\rho_L)$ for a six-blade turbine ($D/T = 1/3$) in sodium sulfate solution (0.4 kg mole/m^3) is 75% of the corresponding value in water. Therefore, P_g/P_o in the sodium sulfate solution ranges from 80 to 90% of the corresponding value in water, the presence of an ionic solute having the greatest effect on the change in P_g/P_o at relatively low values of QN^2 where the difference in gas holdup between the electrolyte solution and water is most pronounced (Figure 5). Comparison of Figures 5 and 6 shows that six-blade turbines and six-blade paddles of the same scale ($D/T = 1/3$) produce an essentially identical gas holdup in nonelectrolytic aqueous systems at constant QN^2/σ. Given that ρ_D/ρ_L in nonelectrolytes is essentially the same for a six-blade turbine or paddle of identical D/T ratio, Figures 2 and 3 for constant N_{We} and N_A show that compared to a six-blade turbine a six-blade paddle exhibits a greater P_g/P_o ratio: 3.35 and 46.4% greater in the small and the large tank, respectively. For specified D, N, and ρ_L, P_o is directly proportional to N_P [Equation (3)]; for $D/T = 1/3$, $W_B/T = 0.1$, and in the turbulent flow regime $N_P = 6.0$ for a six-blade turbine (Rushton et al., 1950) and $N_P = 4.2$ for a six-blade paddle of $W_I/D = 0.20$ (O'Connell and Mack, 1950). Therefore, depending on tank size, P_g for the six-blade paddle ranges from 72.5 (small tank) to 102 (large tank) % of the coresponding P_g for the turbine.

Primarily because of its greater diameter, the four-blade paddle ($D/T = 2/3$) produces a greater gas holdup in nonelectrolyte solutions than either the six-blade turbine or paddle of $D/T = 1/3$ at a specified QN^2/σ as may be seen by comparing Figure 7 with Figure 5 or 6.

A substantial scatter of the data is evident on Figures 5 to 7. As shown by Equation (12), the calculation of ϕ from h_L and h_D is quite sensitive to relatively small errors in measurement of h_L and/or h_D. On the other hand, $\rho_D/\rho_L = h_L/h_D$ is less sensitive than ϕ to measurement errors in h_L and h_D. ρ_D/ρ_L in the power correlations of Figures 2 to 4 was obtained directly from the measured h_L/h_D ratio and was not computed from ϕ as given by Figures 5 to 7. Figures 2 to 4, then, can be used with considerable confidence to predict P_g/P_o if ρ_D/ρ_L is known with reasonable accuracy. We could have chosen to show ρ_D/ρ_L correlated with QN^2/σ; these plots have only relatively minor scatter of the data about the least squares correlating line. However, we chose to show the variation in ϕ despite the increased scatter in order to facilitate direct comparison of our results with those of previous workers. Our gas holdup results for the six-blade turbine in water or aqueous solutions of nonelectrolytes generally are in good agreement with those of Calderbank (1958) and Rushton and Bimbinet (1968). Calderbank found $\phi \propto P_g^{0.4}Q^{0.5}\sigma^{-0.6}$ for a six-blade turbine ($D/T = 1/3$); combining this with our results which show P_g varying as N^3, the Calderbank expression becomes $\phi \propto N^{1.2}Q^{0.5}\sigma^{-0.6}$. Our data obtained over wider ranges of N and Q show that $\phi \propto N^{1.14}(Q/\sigma)^{0.57}$ (Table 5).

Applying the Correlations

The dependency of gas holdup (ϕ) upon the measured mechanical agitation power input to a gas-liquid dispersion (P_g), such as the correlation given by Calderbank (1958), is certainly of general interest. However, as P_g itself is dependent upon ϕ, correlations of this type cannot be used directly to predict P_g for a particular system of specified impeller type, geometrical configuration, and liquid phase physicochemical properties. As previously described, our approach enables ϕ or ρ_L/ρ_D to be determined from correlations in the form of Equation (2) involving only Q, N, and σ for specified impeller type, geometric ratios, and, for the moment, aqueous phase electrolytic character (we plan eventually to generalize the holdup correlations to explicitly include the effect of ionic strength). The predicted ρ_L/ρ_D is then used with the correlation in the form of Equation (1) which is applicable to the particular system to compute the power ratio P_g/P_o in the turbulent flow regime. Finally, P_g is obtained from the latter ratio and the value of P_o at the same N computed from power number correlations for the ungassed liquid [Equation (4)].

Our correlations provide a basis for a generalized approach which, however, must be extended by further work before possibly becoming universally applicable to different tank sizes. Our correlations incorporate the volume-average ρ_D/ρ_L; however, the density ratio in the vicinity of the impeller is more likely to be the parameter pertinent to the change in power which results from sparging a mechanically agitated liquid. Use of the latter density ratio could improve the correlations, but the use of the volume-average value is necessary now since it is the only readily measurable density ratio.

Interchange of Correlations

The experimental results are summarized in Table 5, which lists the best-fit values of C_1 and m in Equation (1) and C_2 and z in Equation (2) for the various impellers and aqueous systems investigated. As previously discussed, exponent n in Equation (1) was found to be a constant ($n = -0.38$) for all impeller-aqueous system combinations.

Table 5 shows that P_g/P_o at specified N_{We}, N_A, and V is highly dependent on the type of impeller used to mechanically agitate a gas dispersion in which the liquid phase consists of water or aqueous solutions of nonelectrolytes.

The experimental data of Clark and Vermeulen (1963) for different impellers also strongly indicate that the power reduction criteria for sparged systems are specific to the particular impeller configuration used. Their generalized correlation was only attainable through the use of a dimensionless geometric group that incorporated the impeller-to-tank-volume ratio

$$G_v = [(D^2 \cdot W_I)/(T^2 \cdot h_L)] \qquad (13)$$

However, their correlation was developed from data obtained using only four-blade impellers and does not appear to be applicable to six-blade impeller systems.

It is clearly evident that P_g for a particular type of impeller cannot be determined accurately from a N_P correlation for that impeller coupled with a P_g/P_o correlation for a different type of impeller, particularly if the D/T ratios for the two impeller types are quire different. For example, Miller (1974) studied the mass transfer characteristics of stirred tanks equipped with four-blade paddles having $D/T = 2/3$. He did not measure P_g/V but computed it from the P_o-N_P correlations of Rushton et al. (1950) or O'Connell and Mack (1950) combined with the P_g-P_o correlation of Michel and Miller (1962) given by Equation (9). The P_g/P_o results of our study (which for the case of the six-blade turbine are in good agreement with those of Michel and Miller) indicate that by interchanging P_g correlations for these two markedly different impellers, Miller (1974) may have overestimated P_g for his four-blade paddles by a factor of 1.3 to 2.1 in his smallest tank and underestimated it by a factor of 0.71 to 0.87 in his intermediate sized tank. The exact magnitude of such possible differences is difficult to estimate, since he applied an empirical correction factor for surface aeration to compute an effective power input rather than using the calculated total mechanical power input on which the specific area and gas holdup correlations of Calderbank (1958) originally were based. Surface aeration is known to affect the mass transfer characteristics of sparged gas-liquid dispersions; possibly, it also affects the gassed power characteristics of such dispersions, but there is no direct experimental evidence to support or refute the latter hypothesis. In any case, such an effect is implicitly included in our power correlations and those of previous workers. Possible errors in computing P_g could then result in errors in the specific interfacial areas and bubble average diameters which Miller (1974) computed, in part, from the correlations of Calderbank (1958) for six-blade turbines ($D/T = 1/3$); in addition, it seems unlikely that Calderbank's correlations are applicable to four-blade paddles ($D/T = 2/3$), even when the correct P_g is used.

For the six-blade turbine at least, P_g/P_o is significantly different for electrolyte solutions compared to the corresponding ratio in nonelectrolytic aqueous phases. Gassed power correlations based on water or aqueous solutions of nonelectrolytes (Calderbank, 1958; Michel and Miller, 1962; Pharamond et al., 1975) do not appear to be applicable to gas dispersions in aqueous electrolyte solutions, even for the same type of impeller and D/T ratio.

ACKNOWLEDGMENT

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NOTATION

C_1 = proportionality constant in Equation (1), dimensionless
C_2 = proportionality constant in Equation (2), $kg \cdot s/m^3)^z$
C_8 = constant in Michel-Miller function, Equation (9), $W^{0.14} s^{0.189} m^{-0.987}$
D = impeller diameter, m
g = exponent on ρ_L/ρ_D, dimensionless
g = gravitational acceleration, m/s^2
g_c = conversion factor, kg m/Kg s^2
G_v = geometrical group, $D^2 W_I/T^2 h_L$, dimensionless
h_D = dispersion height above tank bottom, m
h_L = clear liquid height above tank bottom, m
H_I = height of impeller above tank bottom, m
H_S = height of sparger outlet above tank bottom, m
$K_L a$ = overall volumetric mass transfer coefficient, s^{-1}
L_I = length of impeller blade, m
L_S = distance of sparger from tank centerline, m
m = exponent of Weber number, dimensionless
n = exponent of aeration number, dimensionless

N = impeller rotational sped, s^{-1}
N_A = aeration number, Q/ND^3, dimensionless
N_B = number of impeller blades
N_{Fr} = Froude number, N^2D/g, dimensionless
N_P = power number, $P_o g_c/\rho_L N^3 D^5$, dimensionless
N_{Re} = impeller Reynolds number, $D^2 N \rho_L/\mu$, dimensionless
N_{We} = impeller Weber number, $N^2 D^3 \rho_L/\sigma$, dimensionless
P_g = mechanical agitation power in gas-liquid dispersion, W
P_o = mechanical agitation power in ungassed liquid, W (Kg m/s in N_P)
P_q = sparge gas isothermal expansion power ($P_q = \rho_L g h_L Q$), W
Q = volumetric gas sparging rate (30°C, $1.013 \cdot 10^5$ N/m^2), m^3/s
T = tank internal diameter, m
T_I = impeller blade thickness, m
V = liquid volume, m^3
W_B = baffle width, m
W_I = impeller blade width, m
z = exponent in Equation (2), dimensionless
ϕ = gas holdup volume fraction, dimensionless
μ = liquid phase viscosity, $N \cdot s/m^2$
ρ = mass density, kg/m^3
σ = air-liquid surface tension, N/m

Subscripts

D = property of gas-liquid dispersion
L = property of liquid

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Degrees of Anisotropy for Fluid Flow and Diffusion (Electrical Conduction) Through Anisotropic Porous Media

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This work involves a quantitative comparison between diffusion (or electrical conduction) and fluid flow occurring in the principal directions of certain simple anisotropic porous media, namely, clusters of parallel circular (or elliptic) cylindrical fibers. The degrees to which the observed macroscopic properties, permeability, and effective diffusivity (electrical conductivity) depend upon the direction of flow are determined analytically. Systems of parallel fibers are shown to be considerably more anisotropic with respect to fluid flow than with respect to diffusion (or electrical conduction), this effect being very pronounced with flattish fibers. It would not appear that any general analogy exists between diffusion (electrical conduction) and fluid flow occurring within anisotropic porous media.

SCOPE

Many porous media encountered in practice, especially those formed as a result of industrial or geological sedimentation processes, are anisotropic in constitution, for example, filter beds, river beds, drilling mud cakes, un-